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EVALUATION OF THE DETERMINANT OF IDENTIFICATION EQUATIONS FOR A LINEAR MODEL OF A MECHANICAL VIBRATORY SYSTEM†

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This study was motivated by investigations of the dynamic characteristics of high-speed rotors in hydrostatic bearings, conducted at the University of Poitiers, France. The model is defined by linear ordinary differential equations with undetermined coefficients (reduced masses and damping and elasticity coefficients, which determine the action of a fluid film of an annular seal on the rotor). An identification system of algebraic equations is set up based on tests of the system. By a test we mean the excitation (under certain initial conditions) of a special form of vibratory mode of motion in the system being modelled, by the application of external forces. Measurements are made of the positions of the rotor at each instant of time over a given time interval, and of the forces exerted by the fluid on the rotor. For the identification system to be solvable, it is necessary, in particular, for its determinant to be non-zero. Analytical expressions are obtained for the determinant of the identification system for a model with an arbitrary number of degrees of freedom and for special cases of models with one and two degrees of freedom, with and without damping. A time-domain method of identification is employed. The determinant is evaluated for sinusoidal test vibrations. Such motions correspond to forced or natural vibrations of the system being modelled. In the simplest cases the determinant can be factorized, which leads to simple rules for choosing tests: the minimum number of tests necessary for identification equals twice the number of degrees of freedom in the model; not all the frequencies of the vibrations should be the same; if the frequencies are the same, the corresponding vectors of vibration amplitudes must be linearly independent in the configuration space.

Two basic methods of identification, based on two methods of measurement, have been considered in the literature. The first, the frequency-domain method, is based on Fourier transforms and measurement of the frequency characteristics of steady forced vibrations of the system being modelled [1, 2]. The second, the time-domain method, which has recently seen considerable progress, relies on new high-precision measurement equipment, which is capable of measuring the parameters of motion as practically continuous functions of time [3–6]. A recent review of identification methods [7], which is far from complete, devotes much attention, apart from the mathematical aspects of the approach, to methods for exciting test vibrations. It has been shown [8] that increasing the number of excitation frequencies reduces the effect of noise. A frequency-domain method [9], based on harmonic test vibrations [10] and measurement of the spectral energy density of the vibrations on a special test rig [11], has been used to identify a model of high-speed hybrid bearings [11]. Most of the identification methods are frequency-domain methods. Recent publications, however, have shown that time-domain methods are simpler and more effective. Most interesting in this regard are papers [12, 13] in which the time-domain approach is combined with the method of least squares and it is shown that the necessary number of tests is less than in frequency-domain methods.

1. ILLUSTRATION OF THE IDENTIFICATION METHOD FOR A SYSTEM OF FIRST-ORDER LINEAR EQUATIONS

To simplify the presentation of the time-domain method of identification, we will first consider a canonical system of first-order differential equations

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$$\dot{z} = Pz \tag{1.1}$$

where P is an $n \times n$ matrix of unknown coefficients, z is an n-dimensional phase column-vector (the vector of deviations of the generalized coordinates and velocities from the values in steady motion) and t is the time; a dot denotes differentiation with respect to t.

Present-day measuring equipment yields numerical values of the generalized coordinates and applied forces as continuous functions of time. Suppose s tests have been performed, producing s measurements of the phase vector $z_i(t)$, $t \in [0, T_i]$ $(i = 1, \dots, s)$. To determine the elements of the matrix P one uses the method of least squares with the functional

$$S(P) = \frac{1}{2}\sum_{i=1}^{s}\int_0^{T_i}\|\dot{z}_i(t) - Pz_i(t)\|^2 dt = \frac{1}{2}\mathrm{tr}(P^T PZ) - \mathrm{tr}(PF^T) + \dots \tag{1.2}$$

$$Z = \sum_{i=1}^{s}\int_0^{T_i} z_i z_i^T dt, \quad F = \sum_{i=1}^{s}\int_0^{T_i} z_i \dot{z}_i^T dt \tag{1.3}$$

where the dots stand for terms independent of P, the superscript T denotes transposition, and the symbols $\|\cdot\|$ denote the Euclidean norm of a vector. In what follows the symbols $\|\cdot\|$ will be used to "frame" matrices, specified by their numerical or block elements. The condition for a minimum

$$\mathrm{grad}S(P) = PZ - F = 0$$

yields a linear identification equation for determining the matrix P:

$$PZ = F \tag{1.4}$$

Note that this equation can be obtained by tensor multiplication of the equation of motion (1.1) by z_i, integration with respect to $t \in [0, T_i]$ and summation over all the tests. For the equation to be solvable, the determinant of the matrix Z must not vanish. Starting from definition (1.3) of Z, we can conclude only that it is symmetric and non-negative. The latter conclusion follows from the relation

$$y^T Z y = \sum_{i=1}^{s}\int_0^{T_i}(z_i^T y)^2 dt \geq 0 \Rightarrow \det Z \geq 0$$

Let us evaluate the determinant $\det Z$, using the natural vibrations of the system as tests

$$z_i(t) = c_i \exp\lambda_i t, \quad t \in [0, T_i], \quad i = 1, \dots, s \tag{1.5}$$

where λ_i and c_i are the eigenvalues and complex phase eigenvectors of system (1.1). Then the matrix Z is

$$Z = \sum_{i=1}^{s}\mu_i^2 c_i c_i^T = BB^T; \quad B = \|\mu_1 c_1, \dots \mu_s c_s\|, \quad \mu_i = \sqrt{\int_0^{T_i}\exp 2\lambda_i t dt}$$

The indeterminacy in the value of μ_i is unimportant, since the expression for $\det Z$ involves only μ_i^2. If $s < n$, the matrix B may be completed by $n - s$ columns of zeroes to a square matrix $\bar{B}$:

$$Z = \bar{B}\bar{B}^T; \quad \bar{B} = \|\mu_1 c_1, \dots, \mu_s c_s, 0, \dots, 0\| \Rightarrow \det Z = (\det\bar{B})^2 = 0 \tag{1.6}$$

By last relation in (1.6), a necessary condition for Z to be non-singular is that $s \geq n$. In what follows, we will confine our attention to the case $s = n$. Then

$$\det Z = \mu_1^2 \dots \mu_n^2 (\det C)^2, \quad C = \|c_1, \dots, c_n\|$$

Thus, a necessary condition for the identification system to be non-degenerate is that the vectors of complex phase amplitudes $c_1, \ldots, c_n$ should be linearly independent. By virtue of the continuity of det$Z$ as a function of the test vibration parameters (1.5), the identification system (1.4) will remain non-degenerate even if the test vibrations are only close to the natural ones.

2. THE SYSTEM OF SECOND-ORDER EQUATIONS WITHOUT DAMPING

A disadvantage of the approach described in the previous section (apart from the difficulty of working with complex amplitudes) is that it is impossible to consider external applied forces and forced vibrations of the system. The term corresponding to an applied force, if simply added on the right of Eq. (1.1), would have the sense of a force referred to the reduced mass. Such a force cannot be measured by instruments, since the reduced mass is not known in advance.

Let us consider the equation of vibrations of a linear mechanical system with n degrees of freedom and explicitly written reduced mass, "damping" and "elasticity coefficients".

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{2.1}$$

Equation (2.1) with $c = 0$ may be rewritten as

$$Qu = f(t); \quad u = \|\ddot{x}^T, x^T\|^T, \quad Q = \|m, k\| \tag{2.2}$$

where a compound $2n$-vector u and compound $n \times 2n$ matrix Q have been introduced.

Let us consider s tests $u_i(t)$, $t \in [0, T_i]$ $(i = 1, \ldots, s)$. As in the previous section, the identification equations are obtained by tensor multiplication of Eq. (2.2) by u_i, integration with respect to $t \in [0, T_i]$ and summation over $i = 1, \ldots, s$

$$QU = G; \; U = \sum_{i=1}^{s} \int_0^{T_i} u_i u_i^T dt, \quad G = \sum_{i=1}^{s} \int_0^{T_i} f_i u_i^T dt \tag{2.3}$$

where the $2n \times 2n$ matrix U is analogous to the matrix Z of the previous section and is symmetric and non-negative. To prove that it is non-singular, we must express the structure of the tests more specifically. Now, as tests, besides natural vibrations, one must consider forced vibrations $x_i = a_i\cos(\omega_i t + \varphi_i)$ at frequencies ω_i and real n-dimensional configuration vectors of amplitudes a_i

$$u_i = b_i\cos(\omega_i t + \varphi_i), \quad b_i = \|-\omega_i^2 a_i^T, a_i^T\|^T, \quad i = 1, \ldots, s \tag{2.4}$$

After substituting expressions (2.4) into Eq. (2.3), we get

$$\begin{gathered} U = \sum_{i=1}^{s} b_i b_i^T \gamma_i = \bar{B}\bar{B}^T; \quad \bar{B} = \|\mu_1 b_1, \ldots, \mu_s b_s\|, \quad \mu_i = \sqrt{\gamma_i}, \quad i = 1, \ldots, s \\ \gamma_i = \int_0^{T_i} \cos^2(\omega_i t + \varphi_i)dt = \frac{1}{4\omega_i}(2\omega_i T_i + \sin 2(\omega_i T_i + \varphi_i) - \sin 2\varphi_i) \end{gathered} \tag{2.5}$$

Using formulae (2.5), as in the previous section, we can convince ourselves that det$U = 0$ if $s < 2n$. A necessary condition for U to be non-singular is that $s \geq 2n$ and that the rank of B be $2n$. If $s = 2n$, we have det$U$ = (det$\bar{B}$), and a necessary condition for U to be non-singular is that

$$\det\bar{B} = \mu_1 \ldots \mu_{2n} \det B \neq 0, \quad B = \|b_1, \ldots, b_{2n}\| \tag{2.6}$$

or that all the extended amplitude $2n$-vectors b_i are linearly independent. This condition cannot be satisfied if the vibration frequencies ω_i are the same for all tests, since then the matrix B will be identical rows.

Using Laplace's formula, we expand $\det B$ in terms of the minors of its first n rows

$$\det B = \sum N_{i_1,\dots,i_n}\omega_{i_1}^2\dots\omega_{i_n}^2$$

$$N_{i_1,\dots,i_n} = (-1)^{n+1+2+\dots+n+i_1+\dots+i_n}\det\|a_{i_1},\dots,a_{i_n}\|\det\|a_{j_1},\dots,a_{j_n}\| \tag{2.7}$$

$$\{i_1,\dots,i_n\}\subset\{1,\dots,2n\},\quad \{j_1,\dots,j_n\} = \{1,\dots,2n\}\backslash\{i_1,\dots,i_n\}$$

where the symbol Σ stands for summation over the C_{2n}^n combinations of subscript values $i_1,\dots,i_n$ (arranged in increasing order) in the natural number series $\{1, \dots, 2n\}$, and $j_1, \dots, j_n$ denotes the sequence of subscripts (in increasing order) that complete the sequence $i_1, \dots, i_n$ to the set $\{1, \dots, 2n\}$. Remembering that

$$(-1)^{n+1+2+\dots+n} = (-1)^{[n/2]},\quad (-1)^{i_1+\dots+i_n+j_1+\dots+j_n} = (-1)^{1+\dots+2n} = (-1)^n$$

$$N_{j_1,\dots,j_n} = (-1)^n N_{i_1,\dots,i_n}$$

where $[n/2]$ is the integer part of the number $n/2$, we can rewrite formula (2.7) as

$$\begin{aligned}\det B &= \Sigma' N_{i_1,\dots,i_n}\{\omega_{i_1}^2\dots\omega_{i_n}^2 + (-1)^n\omega_{j_1}^2\dots\omega_{j_n}^2\}\\ N_{i_1,\dots,i_n} &= (-1)^{[n/2]+i_1+\dots+i_n}\det\|a_{i_1},\dots,a_{i_n}\|\det\|a_{j_1},\dots,a_{j_n}\|\end{aligned} \tag{2.8}$$

where Σ' stands for summation over half the combinations of subscripts $i_1, \dots, i_n$ (omitting mutually complementary combinations). As verified by the MAPLE system, in an experiment performed up to an including $n = 4$, formula (2.8) may be rewritten as

$$\det B = \frac{1}{n!2^{n-1}}\Sigma' N_{i_1,\dots,i_n}(\omega_{i_1}^2+\dots+\omega_{i_n}^2-\omega_{j_1}^2-\dots-\omega_{j_n}^2)^n \tag{2.9}$$

In the case of a system with one degree of freedom ($n = 1, s = 2$), formulae (2.8) and (2.9) are identical, and the expression for $\det U$ becomes

$$\det U = \gamma_1\gamma_2(\det B)^2 = a_1^2a_2^2\gamma_1\gamma_2(\omega_1^2-\omega_2^2)^2$$

Hence it follows that for systems with one degree of freedom the vibration frequencies in the two tests must be different.

In the case of a system with two degrees of freedom ($n = 2, s = 4$), formula (2.9) has the form

$$\begin{aligned}\det B &= \frac{1}{2}N_{1,2}(\omega_1^2+\omega_2^2-\omega_3^2-\omega_4^2)^2 + \\ &+\frac{1}{2}N_{1,3}(\omega_1^2+\omega_3^2-\omega_2^2-\omega_4^2)^2+\frac{1}{2}N_{1,4}(\omega_1^2+\omega_4^2-\omega_2^2-\omega_3^2)^2\end{aligned} \tag{2.10}$$

where

$$N_{1,2} = \frac{1}{2}\det\|a_1, a_2\|\det\|a_3, a_4\|$$

$$N_{1,3} = -\frac{1}{2}\det\|a_1,a_3\|\det\|a_2,a_4\|,\quad N_{1,4} = \frac{1}{2}\det\|a_1,a_4\|\det\|a_2,a_3\|$$

In particular, if there are equal frequencies, $\omega_1 = \omega_2$ and $\omega_3 = \omega_4$, the expression for $\det U$ becomes

$$\begin{aligned}\det U &= \gamma_1\gamma_2\gamma_3\gamma_4(\det B)^2 = \\ &= \frac{1}{16}\gamma_1\gamma_2\gamma_3\gamma_4\{\det\|a_1,a_2\|\det\|a_3,a_4\|(\omega_1^2-\omega_3^2)^2\}^2\end{aligned}$$

Thus, in the case of a model with two degrees of freedom without damping, the identification system will be non-degenerate, in particular, if $\omega_1 = \omega_2 \neq \omega_3 = \omega_4$ and the amplitude vectors a_1, a_2 and a_3, a_4 are non-collinear in pairs.

In the case $n = 2$, $s = 4$, formula (2.8) becomes

$$\det B = 2N_{1,2}(\omega_1^2\omega_2^2 + \omega_3^2\omega_4^2) + 2N_{1,3}(\omega_1^2\omega_3^2 + \omega_2^2\omega_4^2) + 2N_{1,4}(\omega_1^2\omega_4^2 + \omega_2^2\omega_3^2) \tag{2.11}$$

For a simple derivation of formula (2.10) from formula (2.11), rewrite the latter in the form

$$\det B = W^T P W; \quad P = \left\| \begin{matrix} 0 & N_{1,2} & N_{1,3} & N_{1,4} \\ N_{1,2} & 0 & N_{1,4} & N_{1,3} \\ N_{1,3} & N_{1,4} & 0 & N_{1,2} \\ N_{1,4} & N_{1,3} & N_{1,2} & 0 \end{matrix} \right\|, \quad W = \left\| \begin{matrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{matrix} \right\|$$

The matrix P is symmetric, and consequently may be diagonalized by an orthogonal transformation. Since its trace is zero, the sum of its eigenvalues is zero. The eigenvalues and eigenvectors are

$$\lambda_1 = N_{1,2} - N_{1,3} - N_{1,4} = 2N_{1,2}, \quad Y_1 = (1, 1, -1, -1)^T$$

$$\lambda_2 = -N_{1,2} + N_{1,3} - N_{1,4} = 2N_{1,3}, \quad Y_2 = (1, -1, 1, -1)^T$$

$$\lambda_3 = -N_{1,2} - N_{1,3} + N_{1,4} = 2N_{1,4}, \quad Y_3 = (1, -1, -1, 1)^T$$

$$\lambda_4 = N_{1,2} + N_{1,3} + N_{1,4} = 0, \quad Y_4 = (1, 1, 1, 1)^T$$

The sum $N_{1,2} + N_{1,3} + N_{1,4}$ corresponds in formula (2.6) to the determinant $\det B$ with equal rows and must therefore vanish. The vectors Y_1, Y_2, Y_3 and Y_4 have the same Euclidean norm 2. In principal axes, the matrix P may be reconstructed as

$$P = \frac{1}{4}(\lambda_1 Y_1 Y_1^T + \lambda_2 Y_2 Y_2^T + \lambda_3 Y_3 Y_3^T + \lambda_4 Y_4 Y_4^T) =$$
$$= \frac{1}{2}(N_{1,2} Y_2 Y_2^T + N_{1,3} Y_3 Y_3^T + N_{1,4} Y_4 Y_4^T)$$

This immediately implies formula (2.10).

We will now present a derivation of formula (2.11) from formula (2.10). Expanding the first square in formula (2.10), we group the terms as follows:

$$(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2)^2 = L + 4(\omega_1^2\omega_2^2 + \omega_3^2\omega_4^2)$$

$$L = \omega_1^4 + \omega_2^4 + \omega_3^4 + \omega_4^4 - 2(\omega_1^2\omega_2^2 + \omega_1^2\omega_3^2 + \omega_1^2\omega_4^2 + \omega_2^2\omega_3^2 + \omega_2^2\omega_4^2 + \omega_3^2\omega_4^2)$$

The same term L will occur in the other two squares as well. Summation of the terms with the factor L gives zero, because $N_{1,2} + N_{1,3} + N_{1,4} = 0$. The second (explicitly written) term in the sum yields formula (2.11).

3. THE SYSTEM OF SECOND-ORDER EQUATIONS WITH DAMPING

We now return to the general equation (2.1) for vibrations of a linear mechanical system with n degrees of freedom. For a more convenient evaluation of the determinant, we will change the order of the terms in Eq. (2.1), introducing a compound $3n$-vector υ and a compound $n \times 3n$ matrix R,

$$R\upsilon = f(t); \quad \upsilon = \|\dot{x}^T, x^T, \dot{x}^T\|^T, \quad R = \|m, k, c\| \tag{3.1}$$

Consider s tests $v_i(t)$, $t \in [0, T_i]$ $(i = 1, \dots, s)$. The identification equations for determining the matrix R are obtained by tensor multiplication of Eq. (3.1) by v_i, integration with respect to $t \in [0, T_i]$, and summation over $i = 1, \dots, s$

$$RV = H; \quad V = \sum_{i=1}^{s} \int_0^{T_i} v_i v_i^T dt, \quad H = \sum_{i=1}^{s} \int_0^{T_i} f_i v_i^T dt \tag{3.2}$$

where the $3n \times 3n$ matrix V is analogous to the matrix U of the previous section and is symmetric and non-negative. To specify the structure of the tests in more detail; we put $x_i = a_i \cos(\omega_i t + \varphi_i)$, and then

$$v_i = \left\| -a_i^T \omega_i^2 \cos(\omega_i t + \varphi_i),\ a_i^T \cos(\omega_i t + \varphi_i),\ -a_i^T \omega_i \sin(\omega_i t + \varphi_i) \right\|^T \tag{3.3}$$

After substituting expression (3.3) into Eq. (3.2), we get

$$V = \sum_{i=1}^{s} \left\| \begin{matrix} a_i a_i^T \omega_i^4 \gamma_i & -a_i a_i^T \omega_i^2 \gamma_i & -a_i a_i^T \omega_i^3 \alpha_i \\ -a_i a_i^T \omega_i^2 \gamma_i & a_i a_i^T \gamma_i & a_i a_i^T \omega_i \alpha_i \\ -a_i a_i^T \omega_i^3 \alpha_i & a_i a_i^T \omega_i \alpha_i & a_i a_i^T \omega_i^2 \beta_i \end{matrix} \right\| =$$

$$= \left\| \begin{array}{c|c} U & \begin{matrix} -\Sigma a_i a_i^T \omega_i^3 \alpha_i \\ \Sigma a_i a_i^T \omega_i \alpha_i \end{matrix} \\ \hline -\Sigma a_i a_i^T \omega_i^3 \alpha_i \quad \Sigma a_i a_i^T \omega_i \alpha_i & DD^T \end{array} \right\| \tag{3.4}$$

where

$$D = (\omega_1 \nu_1 a_1, \dots, \omega_s \nu_s a_s), \quad \nu_i = \sqrt{\beta_i}$$

$$\beta_i = \int_0^{T_i} \sin^2(\omega_i t + \varphi_i) dt = \frac{1}{4\omega_i}(2\omega_i T_i - \sin 2(\omega_i T_i + \varphi_i) + \sin 2\varphi_i)$$

$$\alpha_i = \int_0^{T_i} \sin(\omega_i t + \varphi_i)\cos(\omega_i t + \varphi_i) dt = -\frac{1}{4\omega_i}(\cos 2(\omega_i T_i + \varphi_i) - \cos 2\varphi_i)$$

Note that the integrals γ_i and β_i are always positive and their expressions contain terms linear in T_i with positive coefficients, while the integral α_i is a bounded periodic function of T_i. Assuming that the positive numbers T_i are sufficiently large and that $s \geq 2n$, we express $\det V$ as a polynomial in powers of γ_i, β_i. The maximum degree $3n$ will be that of the term $\det U \det(DD^T)$; the degree of the remaining terms will be at most $3n - 2$; $\det(DD^T) \neq 0$ if the matrix D is of rank n, which is always true when the matrix B is of rank $2n$.

Thus, for sufficiently long test times, the conditions for the matrix U to be non-singular will be sufficient conditions for the matrix V to be non-singular, hence also sufficient conditions for the identification system of equations to be non-degenerate.

In the case of a system with one degree of freedom and damping, and with two tests, $n = 1$, $s = 2$, formula (3.4) becomes

$$\det V = a_1^2 a_2^2 \gamma_1 \gamma_2 (\omega_1^2 - \omega_2^2)^2 \left\{ \frac{a_1^2 \omega_1^2}{\gamma_1}(\beta_1 \gamma_1 - \alpha_1^2) + \frac{a_2^2 \omega_2^2}{\gamma_2}(\beta_2 \gamma_2 - \alpha_2^2) \right\}$$

The quantities $\beta_1 \gamma_1 - \alpha_1^2$ and $\beta_2 \gamma_2 - \alpha_2^2$ are always positive, and the only condition necessary for successful identification, as when there is no damping, is that the vibration frequencies in the tests should be different.

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